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I. Introduction

- AdS/CFT duality
  - Type IIb superstring on $AdS_5 \times S^5$, $\mathcal{N} = 4$ SU(N) SYM ($N \to \infty$)

A remarkable feature: an integrable structure behind AdS/CFT.

- It enables us to check analytically the conjectured relations in AdS/CFT.
  - E.g. anomalous dims. of composite ops., Wilson loops ...

Next issue

Integrable deformations of $AdS_5 \times S^5$ superstring
Preserving the integrability while deforming the background in a non-trivial way

- Yang-Baxter sigma model approach [Klimcik, '02, '08]
- Generalizations to type IIb superstring on $AdS_5 \times S^5$ [Delduc-Magro-Vicedo, '13, '14] [Kawaguchi-Yoshida-Matsumoto, '14]
- Here we focus on a q-deformation of $AdS_5 \times S^5$ superstring [Delduc-Magro-Vicedo, '13, '14]

II. q-deformed $AdS_5 \times S^5$ background (review)

- The q-deformed metric (in the string frame) and the NS-NS two-form were derived. [Arutyunov-Borsato-Frolov, '13]
- Significant progress towards the complete SUGRA solution
  - [Lunin-Roiban-Tseytlin, '14] [Arutyunov-Borsato-Frolov, '15] [Hoare-Tseytlin, '15]

In the q-deformed $AdS_5$

\[
dS^2_{AdS} = R^2(1 + C^2)^2 \left\{ 1 - \frac{1}{1 - C^2 \sinh^2 \rho} \left( - \cosh^2 \rho \, dt^2 + d\rho^2 \right) + \sinh^2 \rho \left( \frac{dC^2 \cos \zeta \, dz + \sin \zeta \, dc^2}{1 - C^2 \sinh^2 \rho \sin \zeta} \right) \right\}
\]

Defection parameter: \( C \in [0, \infty) \)

- A singularity surface exists at \( \rho_s = \text{arc sinh} \frac{1}{C} \)
- GKP-like string solutions cannot stretch beyond the singularity surface. [Frolov, IST14] [T.K, Yoshida, '14]

"The d.o.f. are confined into the region inside the singularity surface?"

- The causal structure around the singularity surface is very similar to the boundary of the global AdS space.
  - E.g. the proper time from any point to the singularity surface is infinite.
- It is useful to employ a coordinate system which describes the spacetime only inside the singularity surface.
  - Our conjecture

The singularity surface might be treated as a holographic screen

III. A possible holographic setup

- Performing a coordinate transformation:

\[
\frac{\cosh \rho}{\sqrt{1 - C^2 \sinh^2 \rho}} = \cosh \chi, \quad \rho \in [0, \text{arc sinh}(1/C)] \quad \text{is mapped to} \quad \chi \in [0, \infty).
\]

- The deformed geometry enclosed by the singularity surface is described as an appropriate analogue of Poincaré coordinates,

\[
dS^2_{\text{AdS}} = R^2(1 + C^2)^2 \left\{ \frac{dz^2 + d\tau^2}{1 - C^2 \sinh^2 \rho} + \frac{C^2(2 \, dz + d\tau)^2}{1 - C^2 \sinh^2 \rho} + \frac{2 \, dz \times d\tau}{1 - C^2 \sinh^2 \rho} \right\}.
\]

- The singularity surface is now located at \( \zeta = 0 \) (boundary).

IV. Minimal surfaces

i) A circular solution in q-deformed Euclidean AdS \(_5\) [T.K, Yoshida, '14]

- The bc is a circle (radius \( a \)) on the “singularity surface” (\( \zeta = 0 \)).
- The solution is

\[
\begin{align*}
\zeta &= a \tan \sigma, \\
r &= a \cosh \sigma, \\
\varphi_1 &= \tau,
\end{align*}
\]

the induced metric can be rewritten in conformally flat form.

- The classical action is finite,

\[
S = \sqrt{\lambda} \sqrt{1 + C^2} \frac{\arccos[C]}{C}
\]

hence no UV cut-off is needed for the string world-sheet.

- The finite result would come from the finiteness of the space-like proper distance to the singularity surface.

- The q-deformation may be regarded as a UV regularization
  - In the case: \( C < 1 \),

\[
S = -\sqrt{\lambda} + \sqrt{\lambda} \frac{\pi}{2C} + O(C)
\]

- To reproduce the regularized result in the \( C \to 0 \) limit, a cut-off \( \varepsilon \) is needed to keep the boundary away from the singularity surface, then

1) Take \( C \to 0 \) with \( \varepsilon \) fixed

2) Expand in \( \varepsilon \) and consider a Legendre transform [Drukker-Gross-Ooguri, '99]

ii) A cusped solution in q-deformed Euclidean AdS \(_5\) [T.K, Yoshida, work in progress]

- The bc is two lines separated by \( \pi - \phi \) on \( \zeta = 0 \) and \( \phi \) on \( S^5 \).
- As world-sheet coordinates, we choose

\[
\frac{\tau}{b} = \frac{1}{E}, \quad \varphi = \frac{J}{E} \frac{1 + b^2 C^2}{1 + b^2}, \quad \sigma = \frac{\sqrt{b^2 + (1 + C^2)^2}}{b} \varphi,
\]

- The two conserved quantities are

\[
p = \frac{1}{E}, \quad q = \frac{J}{E} \frac{1 + b^2 C^2}{1 + b^2},
\]

- The e.o.m. can be expressed as an elliptic eq. and solved as

\[
\begin{align*}
\cosh^2 \chi &= \frac{1 + b^2 + C^2}{(1 + b^2)(1 + b^2 C^2) \cosh^2 \sigma}, \\
\varphi &= \frac{\pi}{2} + \frac{b^2 + C^2}{b^2 + (1 + C^2)^2} \left\{ - \frac{\varpi}{\lambda} + \pi \left( \frac{\varpi}{\lambda} + \frac{2 C^2}{b^2 + C^2} \cos \lambda \right) \left( \frac{\varpi}{\lambda} + \frac{2 C^2}{b^2 + C^2} \right) \right\},
\end{align*}
\]

- The Euclidean action is calculated as

\[
S_{\text{Eucl}} = S_0(C, \varepsilon) + \frac{4 \sqrt{\lambda} b \sqrt{b^2 + 1 + b^2 C^2}}{\pi} \left( -2 \frac{K([k^2 + F (\arcsinh x_0)] k^2)}{x_0} \right).
\]

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\]

- The conformal symmetry is broken

- UV / IR mixing?

- Potentials in the q-deformed case can be determined in terms of \( \phi \) and \( \theta \).

Undetermined case: [Drukker-Forni, '11]

- In the \( \phi \to \pi \) limit, the curves approach antiparallel lines

\[
S_{\text{Sadow}} = S_0(\varepsilon) + \frac{\sqrt{\lambda}}{4 \pi} \left( \frac{b^2 + C^2}{b^2 + (1 + C^2)^2} \right) \left( \frac{\pi}{2} - \varphi \right) \left( \frac{\varphi}{\lambda} + \frac{2 C^2}{b^2 + C^2} \right) \left( \frac{\varphi}{\lambda} + \frac{2 C^2}{b^2 + C^2} \right).
\]

- Taking the \( C \to 0 \) limit first with \( \varepsilon \) fixed, the action is reduced to

\[
S_{\text{Sadow}} = S_0(\varepsilon) - \frac{\sqrt{\lambda}}{4 \pi} \left( \frac{b^2 + C^2}{b^2 + (1 + C^2)^2} \right) \left( \frac{\pi}{2} - \varphi \right) \left( \frac{\varphi}{\lambda} + \frac{2 C^2}{b^2 + C^2} \right) \left( \frac{\varphi}{\lambda} + \frac{2 C^2}{b^2 + C^2} \right).
\]