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On the filamentation instability in degenerate relativistic plasmas[☆]

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ABSTRACT

The filamentation instability of the electromagnetic (EM) beam in an underdense plasma with high level of degeneracy is examined by means of the momentum equation, continuity equation and Maxwell's equations. It has been demonstrated that the instability develops for weakly as well as strongly relativistic degenerate plasma and arbitrary strong amplitude of EM beams.

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1. Introduction

From observations it is evident that certain class of astronomical sources have extremely high luminosities covering almost the whole range of electromagnetic spectra [1]. Usually, for the internal structures of post-main sequence stars, with dense magnetospheric plasma [2], the convenient approach is not working any more because of the high degeneracy of electrons. If this is the case, the physical system has to be described by the Fermi–Dirac distribution when the corresponding Fermi energy, ϵ_F exceeds that of the binding energy of electrons, which leads to ionisation of atoms. For such a system the concentration of electrons is of the order of $10^{26-34} \text{ cm}^{-3}$ and consequently the mean spacing between particles is small compared to the thermal de Broglie wavelength. [3].

One can straightforwardly show that for electron concentrations, n , exceeding the critical value $n_c = m_e^3 c^3 / 3\pi^2 \hbar^3 = 5.9 \times 10^{29} \text{ cm}^{-3}$, the energy of Fermi level $\epsilon_F = m_e c^2 (\gamma_F - 1)$, might exceed the rest mass energy, implying that at such densities, the degenerate electron gas must be treated relativistically even if its “temperature” is nonrelativistic, or even zero. Here, $\gamma_F = \sqrt{1 + p_F^2 / m_e^2 c^2}$ and $p_F = m_e c (n/n_c)^{1/3}$ represent the Fermi relativistic factor and relativistic momentum respectively [4].

In a series of papers [5–8], where authors examined the nonlinear character of interactions of plasma waves and high frequency EM waves it was found that regardless of the degeneracy stable

localised EM structures are induced. The stimulated Raman scattering (SRS) instability for such a plasma was studied in one dimensional case [9]. The authors have shown that the degenerate relativistic plasma reveals interesting properties of SRS instability in low density plasma with the frequency $\omega > 2\omega_e$, where $\omega_e = (4\pi e^2 n_0 / m_e)^{1/2}$ represents the plasma frequency and n_0 is equilibrium number density of electrons. For highly dense plasma, the radiation process might potentially lead to hard X-rays, with specific observational features.

Intense EM waves may undergo filamentation instability (FI) in the underdense plasma. This process inevitably leads to break up of the EM field into multiple beamlets in the direction perpendicular to the incoming radiation [10]. The linear as well as nonlinear regime of FI has been actively studied in nonrelativistic and relativistic intense EM beams [11–15]. However, to the best of our knowledge, the FI of EM radiation in degenerate relativistic plasma is not addressed so far.

In the present paper, we apply the fluid-Maxwell model developed in [5], [7] to study the possibility of FI of intense narrow electromagnetic pulse $L_\perp \ll L_\parallel$ (where L_\parallel and L_\perp are the characteristic longitudinal and transverse spatial dimensions of the field, respectively) in the transparent degenerate electron plasma to show the possibility of FI in relativistic degenerate plasma embedded in the field of arbitrary strong EM radiation.

2. Main consideration

Our approach is based on methods and tools developed in previous works. In particular, we imply the Maxwell's equations and the relativistic electron plasma fluid model. Throughout the

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paper it is assumed that the thermal energy of electrons is negligible compared to their Fermi energy. In the framework of the model the ions are considered to be in stationary states, forming neutralizing background. For zero generalized vorticity $\mathbf{\Omega} = \nabla \times (G\mathbf{p} - e\mathbf{A}/c) = 0$, the electron fluid equations are given by (see Ref. [5] for details):

$$\frac{\partial}{\partial t} (G\mathbf{p} - e\mathbf{A}/c) + \nabla (m_e c^2 G\gamma - e\varphi) = 0, \quad (1)$$

$$\frac{\partial}{\partial t} N + \nabla \cdot (N\mathbf{V}) = 0. \quad (2)$$

The corresponding Maxwell's equations in the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ are expressed as follows:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - c^2 \Delta \mathbf{A} + c \frac{\partial}{\partial t} (\nabla \varphi) + 4\pi e c N \mathbf{V} = 0 \quad (3)$$

$$\Delta \varphi = 4\pi e (N - n_0), \quad (4)$$

where \mathbf{A} and φ represent the EM field vector and scalar potentials respectively; $\mathbf{p} = m_e \gamma \mathbf{V}$ denotes the hydrodynamic momentum of electrons, \mathbf{V} is the velocity and $\gamma = (1 + p^2/m_e^2 c^2)^{1/2}$ is the Lorentz factor; N denotes the electron number density in the laboratory frame of reference, that is related to the rest frame electron density by following $N = \gamma n$. It is evident from Eq. (1) that for the arbitrary strength of relativity defined by the ratio n/n_c the "effective mass" factor G for fully degenerate plasma (i.e. having zero temperature) coincides with the Fermi relativistic factor $G = \gamma_F = (1 + (n/n_c)^{2/3})^{1/2}$.

To study the problem of the nonlinear self-guiding of the EM beam in a highly transparent electron plasma we apply Eqs. (1)–(4), which for the generalized momentum $\mathbf{\Pi} = G\mathbf{p}$ and relativistic factor $\Gamma = G\gamma$ reduce to the following set of dimensionless equations

$$\frac{\partial}{\partial t} (\mathbf{\Pi} - \mathbf{A}) + \nabla (\Gamma - \varphi) = 0, \quad (5)$$

$$\frac{\partial}{\partial t} N + \nabla \cdot \mathbf{J} = 0, \quad (6)$$

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \Delta \mathbf{A} + \frac{\partial}{\partial t} (\nabla \varphi) + \varepsilon^2 \mathbf{J} = 0, \quad (7)$$

$$\Delta \varphi = N - 1, \quad (8)$$

where $\mathbf{J} = N\mathbf{\Pi}/\Gamma$ and $\Gamma = (G^2 + \mathbf{\Pi}^2)^{1/2}$, $\tilde{t} = \omega t$, $\tilde{\mathbf{r}} = \omega \mathbf{r}/c$, $\tilde{\mathbf{A}} = e\mathbf{A}/m_e c^2$, $\tilde{\varphi} = e\varphi/m_e c^2$, $\tilde{\mathbf{\Pi}} = \mathbf{\Pi}/m_e c$, $\tilde{n} = n/n_0$ and $\tilde{N} = N/n_0$ (in the above equations the tilde is omitted). Here we assume that the plasma is highly underdense, $\varepsilon = \omega_e/\omega \ll 1$, where ε is a small parameter of the system and $G = (1 + R_0^2 n^{2/3})^{1/2}$ where $R_0 = (n_0/n_c)^{1/3}$.

Following the method of multiple scale expansion of the equations in the small parameter ε [16], [17], the physical variables ($Q = A, \varphi, \mathbf{\Pi}, \Gamma, N, G$) expand as

$$Q = Q_{\{0\}}(\xi, x_1, y_1, z_2) + \varepsilon Q_{\{1\}}(\xi, x_1, y_1, z_2), \quad (9)$$

where $(x_1, y_1, z_2) = (\varepsilon x, \varepsilon y, \varepsilon^2 z)$ and $\xi = z - bt$ and $(b^2 - 1) \sim \varepsilon^2$. In the framework of the paper we assume that the EM field is circularly polarized

$$\mathbf{A}_{\{0\perp\}} = \frac{1}{2} (\hat{\mathbf{x}} + i\hat{\mathbf{y}}) A \exp(i\xi/b), \quad (10)$$

with a slowly varying function A .

In the zeroth order approximation for the longitudinal (to the direction of EM wave propagation z) component of Eq. (5) one can straightforwardly show $\Pi_{\{z0\}} = 0$ whereas for the transverse

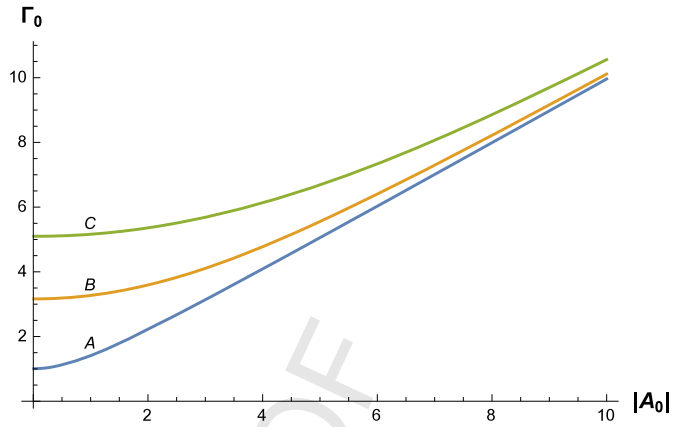


Fig. 1. Dependence of Γ_0 on the EM field strength A_0 for different level of degeneracy: (A) $R_0 = 0.1$, (B) $R_0 = 3$, and (C) $R_0 = 5$.

component we obtain $\mathbf{\Pi}_{\{0\perp\}} = -\mathbf{A}_{\{0\perp\}}$. From the transverse part of Eq. (5) to the first order in ε and Eq. (6) one can conclude that $\Gamma_{\{0\}}$, $\varphi_{\{0\}}$ and $N_{\{0\}}$ are independent from the fast variable ξ . For the transverse part of the first order of Eq. (5) we arrive at

$$-b \frac{\partial}{\partial \xi} [\mathbf{\Pi}_{\perp\{1\}} - \mathbf{A}_{\perp\{1\}}] + \nabla_{\perp} (\Gamma_{\{0\}} - \varphi_{\{0\}}) = 0, \quad (11)$$

where $\nabla_{\perp} = (\hat{\mathbf{x}}\partial/\partial x_1 + \hat{\mathbf{y}}\partial/\partial y_1)$. By averaging Eq. (8) over the fast variable ξ the equation reduces to

$$\nabla_{\perp} \Gamma_{\{0\}} = \nabla_{\perp} \varphi_{\{0\}}, \quad (12)$$

where $\Gamma_{\{0\}} = (G_{\{0\}}^2 + |A|^2)^{1/2}$, $G_{\{0\}} = (1 + R_0^2 n_{\{0\}}^{2/3})^{1/2}$ and the rest frame electron density is denoted by $n_{\{0\}}$. This value is related to the lab frame density by the following relation

$$n_{\{0\}} = \frac{G_{\{0\}} N_{\{0\}}}{\Gamma_{\{0\}}}. \quad (13)$$

Eq. (12) leads to $\Gamma_{\{0\}} - \varphi_{\{0\}} = \Gamma_0$ provided that EM field intensity forms a constant background $|A|^2 \rightarrow |A_0|^2$ for $|r_{\perp}| \rightarrow \infty$ while $\varphi_{\{0\}} \rightarrow 0$ and $N_{\{0\}} \rightarrow 1$. After combining this algebraic relation with Eq. (13) one can obtain the expression for the plasma electron density:

$$N_{\{0\}} = \left(\frac{\Gamma_0}{R_0}\right)^3 \frac{(1 + \Psi) [(1 + \Psi)^2 - (1 + |A|^2)/\Gamma_0^2]^{3/2}}{[(1 + \Psi)^2 - |A|^2/\Gamma_0^2]^{1/2}}, \quad (14)$$

where Ψ denotes the normalized value of φ : $\Psi = \varphi/\Gamma_0$.

An "effective" relativistic factor, Γ_0 , for the given boundary conditions depends on the parameters R_0 and $|A_0|^2$ by the following implicit expression:

$$\left(1 - \frac{1 + |A_0|^2}{\Gamma_0^2}\right) - \frac{R_0^2}{\Gamma_0^2} \left(1 - \frac{|A_0|^2}{\Gamma_0^2}\right)^{1/3} = 0. \quad (15)$$

One can see that when Eq. (15) is satisfied $N_{\{0\}} = 1$ for $\Psi > 0$.

For the weakly degenerate plasma $R_0 \ll 1$ the constant Γ_0 coincides with the Lorentz factor of electrons in classical cold plasmas $\Gamma_0 \simeq (1 + |A_0|^2)^{1/2}$. In the weakly relativistic amplitudes of EM field ($|A_0| \rightarrow 0$) Γ_0 is determined by the degeneracy parameter $\Gamma_0 \simeq (1 + R_0^2)^{1/2}$. In Fig. 1 we show the dependence of Γ_0 on $|A_0|$ for different values of R_0 . In the extreme relativistic amplitudes ($|A_0| \gg 1$), Γ_0 tends to $|A_0|$ (essentially the expression of a nondegenerate plasma) for an arbitrary level of the degeneracy parameter R_0 .

By considering the slowly varying envelope, Maxwell's equation (7) and the Poisson's equation (8) reduce to

$$2i \frac{\partial A}{\partial z_2} + \nabla_{\perp}^2 A + \sigma A - A \frac{N_{(0)}}{\Gamma_0} = 0, \quad (16)$$

$$\nabla_{\perp}^2 \Psi = \frac{1}{\Gamma_0} (N_{(0)} - 1), \quad (17)$$

where $\sigma = (b^2 - 1)/b^2 \varepsilon^2$. For $b = \omega/kc$ one can see that $\sigma - 1/\Gamma_0 = 0$ leads to the following dispersion relation $\omega^2 = k^2 c^2 + \Omega_e^2$, with $\Omega_e = \omega_e/\Gamma_0$ representing a plasma frequency modified by the effective relativistic factor Γ_0 . Introducing the following renormalizations $z = z/\Gamma_0$ and $r_{\perp} = r_{\perp}/\Gamma_0^{1/2}$ Eqs. (16)–(17) can be written as

$$2i \frac{\partial A}{\partial z} + \nabla_{\perp}^2 A + \left(1 - \frac{N}{\Psi + 1}\right) A = 0, \quad (18)$$

$$\nabla_{\perp}^2 \Psi + 1 - N = 0, \quad (19)$$

where we have omitted subscripts for the variables ($z_2, x_1, y_1, N_{(0)}$).

To study the stability of the ground state solution we linearized the system of equations (18)–(19) in the form $A = A_0 + \delta A$, $\Psi = \delta \Psi$, $N = 1 + \delta N$. By assuming that all perturbed quantities ($\delta A, \delta \Psi, \delta N$) depend on coordinates as $\exp(\chi z + i\mathbf{k}_{\perp} \mathbf{r}_{\perp})$, the following dispersion relation can be obtained

$$\chi = \frac{1}{2} k_{\perp} \left(\frac{1 + k_{\perp}^2}{(k_{\perp}^2 + C_{\Psi})} |A_0|^2 C_a - k_{\perp}^2 \right)^{1/2} \quad (20)$$

that implies the necessary condition for the instability

$$\frac{1 + k_{\perp}^2}{(k_{\perp}^2 + C_{\Psi})} |A_0|^2 C_a > k_{\perp}^2, \quad (21)$$

where $C_a = -2[\partial N/\partial |A|^2]_{|A|=|A_0|, \Psi=0}$ and $C_{\Psi} = [\partial N/\partial \Psi]_{|A|=|A_0|, \Psi=0}$. From Eq. (15) one can show that variables C_a, C_{Ψ} are both positive and they can be presented as $C_a = Q/R_0^2$ and $C_{\Psi} = 1 + \Gamma_0^2 Q/R_0^2$ where

$$Q = 2\Gamma_0^{-2} (1 - |A_0|^2/\Gamma_0^2)^{-4/3} \left[3/2 + \Gamma_0^2 - (1 + |A_0|^2) \right]. \quad (22)$$

From the structure of Eqs. (20)–(22) it is evident that the perturbations with the transverse wave number $0 < k_{\perp} < k_{lim}$, where k_{lim} is a certain limiting value, are unstable with respect to z . Since Eq. (20) contains several parameters $|A_0|, \Gamma_0$ and R_0 , which are interrelated by Eq. (15), in general, should be obtained numerically. However, for small amplitude pump waves $|A_0| \ll 1$, $Q \simeq (1 + 2\Gamma_0^2)/\Gamma_0^2$ the dispersion relation reads as

$$\chi = \frac{1}{2} k_{\perp} \left(|A_0|^2 K_{\Gamma_0}^2 - k_{\perp}^2 \right)^{1/2}, \quad (23)$$

where $K_{\Gamma_0} = [(1 + 2\Gamma_0^2)/3\Gamma_0^4]^{1/2}$ and the Lorentz factor determined by the degeneracy parameter R_0 is given by $\Gamma_0 = (1 + R_0^2)^{1/2}$ (see Eq. (15)). For weakly degenerate plasma ($R_0 \ll 1$) $K_{\Gamma_0} \rightarrow 1$ while in the ultrarelativistic case ($R_0 \gg 1$) $K_{\Gamma_0} \simeq (2/3)^{1/2}/R_0$.

From Eq. (23) it is clear that $k_{lim} = |A_0| K_{\Gamma_0} \ll 1$ and the instability increment χ reaches its maximum $\chi_m = |A_0|^2 K_{\Gamma_0}^2/4$ for the transverse wave vector $k_{\perp m} = 2^{-1/2} |A_0| K_{\Gamma_0}$, corresponding to the characteristic spatial scale of filaments $\Delta_{\perp} = \pi/k_{\perp m}$. It is worth noting that by increasing the value of R_0 the spatial scale of filaments increases as well. The "critical" power of the EM field carried by the filaments could be estimated as $P_c = \pi$

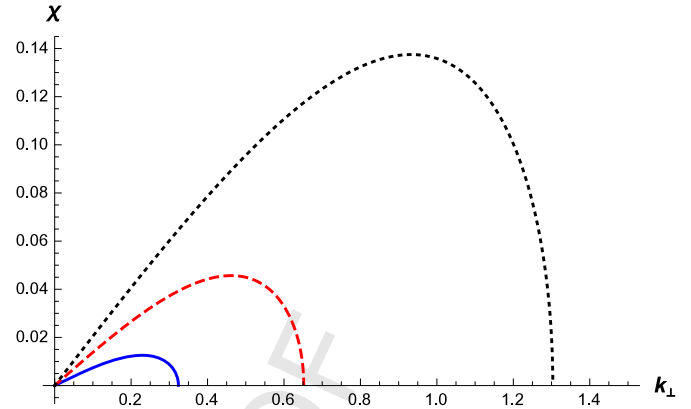


Fig. 2. Dependence of instability increment χ on k_{\perp} for different values of A_0 : $A_0 = 0.5$ – solid line, $A_0 = 1$ – dashed line, $A_0 = 2$ – dotted line.

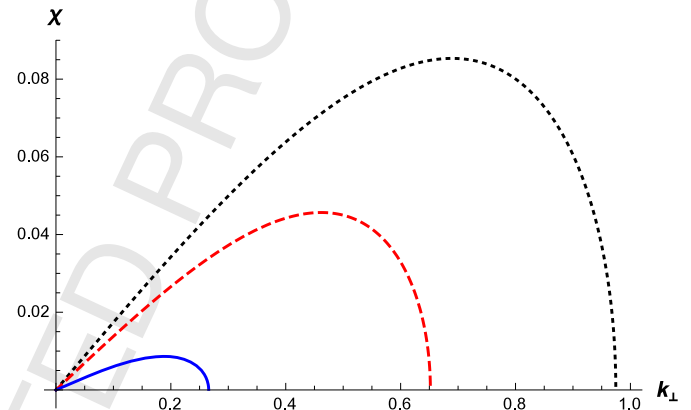


Fig. 3. Dependence of instability increment χ on k_{\perp} for different values of R_0 : $A_0 = 3$ – solid line, $A_0 = 1$ – dashed line, $A_0 = 0.2$ – dotted line.

$|A_0|^2 \Delta_{\perp}^2/4 \simeq 7.8 K_{\Gamma_0}^{-2}$ which, as it is evident, does not depend on the pump strength and increases with R_0 . The mentioned critical power is expressed as $P_{cd} = (m_e^2 c^5/4e^2) (\omega/\omega_e)^2 P_c \approx 1.7 \times 10^{16} K_{\Gamma_0}^{-2} (\omega/\omega_e)^2 \text{erg/sec}$. A physically reasonable interval of allowed densities of the degenerate electron plasma is within $(10^{24} - 10^{34}) \text{ cm}^{-3}$ [$R_0 \sim 1.2 - 25$], leading to the following value of the power $P_{cd} \approx (17 \times 10^{-3} \div 10.6) (\omega/\omega_e)^2 \times 10^{19} \text{ erg/sec}$.

To calculate the increment of FI for the arbitrary strength of EM pump the numerical analyses of Eqs. (15), (20)–(22) has been performed. In Fig. 2 for $R_0 = 1$ we show the dependence of χ on k_{\perp} for different values of $|A_0|$. It is clear that the maximum of the instability increment and the range of k_{\perp} over which an instability occurs ($0 < k_{\perp} < k_{lim}$) increase with increasing values of $|A_0|$. Such a behaviour is valid for the arbitrary strength of degeneracy parameter R_0 . Unlike the regime of weak amplitudes ($|A_0| \ll 1$) in ultrarelativistic case $|A_0| \geq 1$ for the characteristic wave vectors of unstable modes we have $k_m(k_{lim}) \geq 1$. In Fig. 3 we demonstrate dependence of χ on k_{\perp} when $|A_0| = 1$ for the different level of degeneracy R_0 . With increase of the degeneracy level the growth rates and k_{lim} decrease.

It is worth noting that since we deal with X-ray sources the wave amplitude $|A_0|$ is not arbitrarily large in the framework of the developed approach. In particular, $|A_0|$ (in units $eA_0/m_e c^2$) can be given as $|A_0|^2 = 3.65 \times 10^{-12} \times I \times \lambda^2 [\mu]$, where I is the EM wave intensity measured in $\text{erg}/(\text{sec cm}^2)$ and the wave length λ is in micrometres. For $|A_0| = 10$ and $\lambda = 0.1 \text{ nm}$ ($\hbar\omega = 12.4 \text{ KeV}$) $I \simeq 2.7 \times 10^{35} \text{ erg}/(\text{sec m}^2)$ while electric field of the wave is $E \simeq 3 \times 10^{15} \text{ V/cm}$ which is by order of magnitude smaller than the Schwinger limit $E_S = m_e^2 c^3/\hbar e = 1.3 \times 10^{16} \text{ V/cm}$. Above this limit

the EM field is expected to become nonlinear and the considered approach fails. In particular, under these circumstances, the process of electron–positron pair creation can take place.

3. Conclusion

We have shown that the powerful EM beam propagating in the underdense degenerate plasma undergoes the FI. This process takes place regardless of degeneracy, implying that the beam power is larger than the critical power of the appeared structures. The dynamics of formation of filaments in the nonlinear stage and the subsequent complex behaviour should be investigated by means of numerical simulations of Eqs. (18)–(19) and is beyond of the intended scope of the current paper.

The study of the FI driven by extremely strong EM pulses is significant for understanding dynamics of electromagnetic emission originating from a certain class of astrophysical objects. In particular, it has been widely accepted that X-ray emission might appear from accreting white dwarfs (WD) [22]. In this scenario, the mentioned object accretes material from a companion star, resulting in the hitting process of plasma flow on a star's surface and by means of the Bremsstrahlung mechanism the particles decelerate, leading to generation of X-ray radiation. On the other hand, the WDs are composed of highly degenerate electrons and the study of their interaction with the induced X-rays might be very promising. The similar process might take place also in neutron stars (NSs) loaded by accretion disk generating hard X-rays [2]. Interiors of NSs mainly consist of neutrons with approximately 1% of electrons and protons [2], which are also in highly degenerate state, therefore, the FI might be of high significance. Another interesting class of objects where the aforementioned process might develop are the gamma ray bursters (GRB). It is thought that the high energy radiation of GRB might be generated during a supernova explosion of relatively massive stars, which after they collapse form NSs [23]. Since this manuscript was a first attempt of this kind the application of the developed model to the mentioned astrophysical objects is beyond the scope of the paper.

The filamentation instability might be interested in the case of interaction of superstrong electromagnetic radiation with laboratory plasmas [18]. Such plasmas imbedded in the field of the superstrong laser radiation, with intensities of the order of 10^{21-23} W cm⁻² can exhibit various interesting phenomena including self-focusing and FI. The most of the super powerful lasers currently are operating at the wavelength $\lambda \sim 1$ μ m ($\hbar\omega \sim 1.2$ eV). Since the density of degenerate plasma by several orders of magnitude exceeds the critical density for such micron wavelength laser pulses the plasma is opaque. We would like to emphasise that in

the standard scheme of the fast ignition model [19] plasma is supposed to be compressed at temperatures as low as possible, while by means of a powerful ultra short beam the ignition should occur. However, a compressed target can be in degenerate states [20]. Such plasma states can be transparent just for X-ray lasers. Recent achievements in the X-ray free-electron laser technology made it possible to achieve intensities above 10^{20} W cm⁻² at 9.9 keV ($\lambda \sim 1.3 \times 10^{-4}$ μ m, hard X-rays) [21]. This achievement gives us a hope that the increase of X-ray laser pulse intensities are feasible. Effects like FI (self-focusing) can be significant in producing X-rays with small spot sizes in the processes of interaction with highly compressed degenerate plasmas.

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